

1969 AP Calculus BC: Section I

90 Minutes—No Calculator

Note: In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e).

1. The asymptotes of the graph of the parametric equations $x = \frac{1}{t}$, $y = \frac{t}{t+1}$ are
- (A) $x = 0$, $y = 0$ (B) $x = 0$ only (C) $x = -1$, $y = 0$
(D) $x = -1$ only (E) $x = 0$, $y = 1$
-
2. What are the coordinates of the inflection point on the graph of $y = (x+1)\arctan x$?
- (A) $(-1, 0)$ (B) $(0, 0)$ (C) $(0, 1)$ (D) $\left(1, \frac{\pi}{4}\right)$ (E) $\left(1, \frac{\pi}{2}\right)$
-
3. The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between $(0, 0)$ and $(4, 2)$. What are the coordinates of this point?
- (A) $(2, 1)$
(B) $(1, 1)$
(C) $(2, \sqrt{2})$
(D) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$
(E) None of the above
-
4. $\int_0^8 \frac{dx}{\sqrt{1+x}} =$
- (A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6
-
5. If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is
- (A) -2 (B) 0 (C) 2 (D) 4 (E) not defined

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6. What is $\lim_{h \rightarrow 0} \frac{8\left(\frac{1}{2} + h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?
- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.
 (E) It cannot be determined from the information given.
-
7. For what value of k will $x + \frac{k}{x}$ have a relative maximum at $x = -2$?
- (A) -4 (B) -2 (C) 2 (D) 4 (E) None of these
-
8. If $h(x) = f^2(x) - g^2(x)$, $f'(x) = -g(x)$, and $g'(x) = f(x)$, then $h'(x) =$
- (A) 0 (B) 1 (C) $-4f(x)g(x)$
 (D) $(-g(x))^2 - (f(x))^2$ (E) $-2(-g(x) + f(x))$
-
9. The area of the closed region bounded by the polar graph of $r = \sqrt{3 + \cos \theta}$ is given by the integral
- (A) $\int_0^{2\pi} \sqrt{3 + \cos \theta} \, d\theta$ (B) $\int_0^{\pi} \sqrt{3 + \cos \theta} \, d\theta$ (C) $2 \int_0^{\pi/2} (3 + \cos \theta) \, d\theta$
 (D) $\int_0^{\pi} (3 + \cos \theta) \, d\theta$ (E) $2 \int_0^{\pi/2} \sqrt{3 + \cos \theta} \, d\theta$
-
10. $\int_0^1 \frac{x^2}{x^2 + 1} \, dx =$
- (A) $\frac{4 - \pi}{4}$ (B) $\ln 2$ (C) 0 (D) $\frac{1}{2} \ln 2$ (E) $\frac{4 + \pi}{4}$

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11. The point on the curve $x^2 + 2y = 0$ that is nearest the point $\left(0, -\frac{1}{2}\right)$ occurs where y is

- (A) $\frac{1}{2}$
- (B) 0
- (C) $-\frac{1}{2}$
- (D) -1
- (E) none of the above

12. If $F(x) = \int_0^x e^{-t^2} dt$, then $F'(x) =$

- (A) $2xe^{-x^2}$
- (B) $-2xe^{-x^2}$
- (C) $\frac{e^{-x^2+1}}{-x^2+1} - e$
- (D) $e^{-x^2} - 1$
- (E) e^{-x^2}

13. The region bounded by the x -axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \leq x \leq k$ is three times the area of the region for $k \leq x \leq \frac{\pi}{2}$, then $k =$

- (A) $\arcsin\left(\frac{1}{4}\right)$
- (B) $\arcsin\left(\frac{1}{3}\right)$
- (C) $\frac{\pi}{6}$
- (D) $\frac{\pi}{4}$
- (E) $\frac{\pi}{3}$

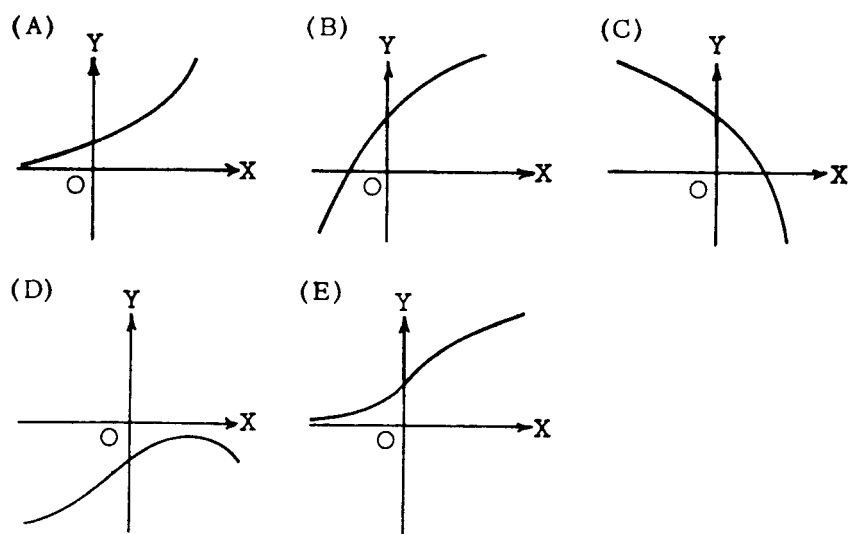
14. If $y = x^2 + 2$ and $u = 2x - 1$, then $\frac{dy}{du} =$

- (A) $\frac{2x^2 - 2x + 4}{(2x - 1)^2}$
- (B) $6x^2 - 2x + 4$
- (C) x^2
- (D) x
- (E) $\frac{1}{x}$

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15. If $f'(x)$ and $g'(x)$ exist and $f'(x) > g'(x)$ for all real x , then the graph of $y = f(x)$ and the graph of $y = g(x)$
- (A) intersect exactly once.
 - (B) intersect no more than once.
 - (C) do not intersect.
 - (D) could intersect more than once.
 - (E) have a common tangent at each point of intersection.

16. If y is a function x such that $y' > 0$ for all x and $y'' < 0$ for all x , which of the following could be part of the graph of $y = f(x)$?



17. The graph of $y = 5x^4 - x^5$ has a point of inflection at

- (A) $(0, 0)$ only
- (B) $(3, 162)$ only
- (C) $(4, 256)$ only
- (D) $(0, 0)$ and $(3, 162)$
- (E) $(0, 0)$ and $(4, 256)$

18. If $f(x) = 2 + |x - 3|$ for all x , then the value of the derivative $f'(x)$ at $x = 3$ is

- (A) -1
- (B) 0
- (C) 1
- (D) 2
- (E) nonexistent

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19. A point moves on the x -axis in such a way that its velocity at time t ($t > 0$) is given by $v = \frac{\ln t}{t}$.

At what value of t does v attain its maximum?

- (A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$
(E) There is no maximum value for v .
-

20. An equation for a tangent to the graph of $y = \arcsin \frac{x}{2}$ at the origin is

- (A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$
(D) $y = 0$ (E) $\pi x - 2y = 0$
-

21. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

- (A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.
-

22. If $f(x) = \int_0^x \frac{1}{\sqrt{t^3 + 2}} dt$, which of the following is FALSE?

- (A) $f(0) = 0$
(B) f is continuous at x for all $x \geq 0$.
(C) $f(1) > 0$
(D) $f'(1) = \frac{1}{\sqrt{3}}$
(E) $f(-1) > 0$
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23. If the graph of $y = f(x)$ contains the point $(0, 2)$, $\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$ and $f(x) > 0$ for all x , then $f(x) =$
- (A) $3 + e^{-x^2}$ (B) $\sqrt{3} + e^{-x}$ (C) $1 + e^{-x}$
- (D) $\sqrt{3 + e^{-x^2}}$ (E) $\sqrt{3 + e^{x^2}}$
-
24. If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x ?
- (A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$
-
25. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x -axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region
- (A) is independent of m .
- (B) increases as m increases.
- (C) decreases as m increases.
- (D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$.
- (E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.
-
26. $\int_0^1 \sqrt{x^2 - 2x + 1} \, dx$ is
- (A) -1
- (B) $-\frac{1}{2}$
- (C) $\frac{1}{2}$
- (D) 1
- (E) none of the above

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27. If $\frac{dy}{dx} = \tan x$, then $y =$

(A) $\frac{1}{2}\tan^2 x + C$

(B) $\sec^2 x + C$

(C) $\ln|\sec x| + C$

(D) $\ln|\cos x| + C$

(E) $\sec x \tan x + C$

28. What is $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\tan x}$?

(A) -1

(B) 0

(C) 1

(D) 2

(E) The limit does not exist.

29. $\int_0^1 (4 - x^2)^{\frac{3}{2}} dx =$

(A) $\frac{2 - \sqrt{3}}{3}$

(B) $\frac{2\sqrt{3} - 3}{4}$

(C) $\frac{\sqrt{3}}{12}$

(D) $\frac{\sqrt{3}}{3}$

(E) $\frac{\sqrt{3}}{2}$

30. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Taylor series about zero for which of the following functions?

(A) $\sin x$

(B) $\cos x$

(C) e^x

(D) e^{-x}

(E) $\ln(1 + x)$

31. If $f'(x) = -f(x)$ and $f(1) = 1$, then $f(x) =$

(A) $\frac{1}{2}e^{-2x+2}$

(B) e^{-x-1}

(C) e^{1-x}

(D) e^{-x}

(E) $-e^x$

32. For what values of x does the series $1 + 2^x + 3^x + 4^x + \cdots + n^x + \cdots$ converge?

(A) No values of x

(B) $x < -1$

(C) $x \geq -1$

(D) $x > -1$

(E) All values of x

33. What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \leq t \leq 2$?

(A) $\frac{11}{4}$

(B) $\frac{7}{2}$

(C) 8

(D) $\frac{33}{4}$

(E) 16

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34. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = \frac{1}{x} + k$ (where k takes all real values)?
- (A) $y = -x$ (B) $y = -x^2$ (C) $y = -\frac{1}{3}x^3$ (D) $y = \frac{1}{3}x^3$ (E) $y = \ln x$
-
35. At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?
- (A) 32 (B) 48 (C) 64 (D) 96 (E) 192
-
36. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is
- (A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24
-
37. Of the following choices of δ , which is the largest that could be used successfully with an arbitrary ε in an epsilon-delta proof of $\lim_{x \rightarrow 2} (1 - 3x) = -5$?
- (A) $\delta = 3\varepsilon$ (B) $\delta = \varepsilon$ (C) $\delta = \frac{\varepsilon}{2}$ (D) $\delta = \frac{\varepsilon}{4}$ (E) $\delta = \frac{\varepsilon}{5}$
-
38. If $f(x) = (x^2 + 1)^{(2-3x)}$, then $f'(1) =$
- (A) $-\frac{1}{2} \ln(8e)$ (B) $-\ln(8e)$ (C) $-\frac{3}{2} \ln(2)$ (D) $-\frac{1}{2}$ (E) $\frac{1}{8}$
-
39. If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?
- (A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

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40. If n is a non-negative integer, then $\int_0^1 x^n dx = \int_0^1 (1-x)^n dx$ for

- (A) no n (B) n even, only (C) n odd, only
(D) nonzero n , only (E) all n

41. If $\begin{cases} f(x) = 8 - x^2 & \text{for } -2 \leq x \leq 2, \\ f(x) = x^2 & \text{elsewhere,} \end{cases}$ then $\int_{-1}^3 f(x) dx$ is a number between

- (A) 0 and 8 (B) 8 and 16 (C) 16 and 24 (D) 24 and 32 (E) 32 and 40

42. If $\int x^2 \cos x dx = f(x) - \int 2x \sin x dx$, then $f(x) =$

- (A) $2 \sin x + 2x \cos x + C$
(B) $x^2 \sin x + C$
(C) $2x \cos x - x^2 \sin x + C$
(D) $4 \cos x - 2x \sin x + C$
(E) $(2 - x^2) \cos x - 4 \sin x + C$

43. Which of the following integrals gives the length of the graph of $y = \tan x$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$?

- (A) $\int_a^b \sqrt{x^2 + \tan^2 x} dx$
(B) $\int_a^b \sqrt{x + \tan x} dx$
(C) $\int_a^b \sqrt{1 + \sec^2 x} dx$
(D) $\int_a^b \sqrt{1 + \tan^2 x} dx$
(E) $\int_a^b \sqrt{1 + \sec^4 x} dx$

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44. If $f''(x) - f'(x) - 2f(x) = 0$, $f'(0) = -2$, and $f(0) = 2$, then $f(1) =$

- (A) $e^2 + e^{-1}$ (B) 1 (C) 0 (D) e^2 (E) $2e^{-1}$
-

45. The complete interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x+1)^k}{k^2}$ is

- (A) $0 < x < 2$ (B) $0 \leq x \leq 2$ (C) $-2 < x \leq 0$
(D) $-2 \leq x < 0$ (E) $-2 \leq x \leq 0$

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|-------|-------|
| 1. B | 24. C |
| 2. C | 25. A |
| 3. B | 26. C |
| 4. D | 27. C |
| 5. E | 28. C |
| 6. B | 29. A |
| 7. D | 30. E |
| 8. B | 31. C |
| 9. C | 32. B |
| 10. E | 33. A |
| 11. B | 34. D |
| 12. A | 35. A |
| 13. C | 36. B |
| 14. E | 37. D |
| 15. B | 38. C |
| 16. B | 39. D |
| 17. B | 40. E |
| 18. E | 41. D |
| 19. C | 42. D |
| 20. A | 43. D |
| 21. B | 44. C |
| 22. E | 45. D |
| 23. C | |

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|-------|-------|
| 1. C | 24. C |
| 2. E | 25. A |
| 3. B | 26. C |
| 4. D | 27. C |
| 5. E | 28. D |
| 6. B | 29. C |
| 7. D | 30. D |
| 8. C | 31. C |
| 9. D | 32. B |
| 10. A | 33. A |
| 11. B | 34. D |
| 12. E | 35. A |
| 13. C | 36. B |
| 14. D | 37. D |
| 15. B | 38. A |
| 16. B | 39. D |
| 17. B | 40. E |
| 18. E | 41. D |
| 19. C | 42. B |
| 20. A | 43. E |
| 21. B | 44. E |
| 22. E | 45. E |
| 23. D | |

1. C For horizontal asymptotes consider the limit as $x \rightarrow \pm\infty$: $t \rightarrow 0 \Rightarrow y = 0$ is an asymptote
For vertical asymptotes consider the limit as $y \rightarrow \pm\infty$: $t \rightarrow -1 \Rightarrow x = -1$ is an asymptote

2. E $y = (x+1)\tan^{-1}x$, $y' = \frac{x+1}{1+x^2} + \tan^{-1}x$

$$y'' = \frac{(1+x^2)(1) - (x+1)(2x)}{(1+x^2)^2} + \frac{1}{1+x^2} = \frac{2-2x}{(1+x^2)^2}$$

y'' changes sign at $x = 1$ only. The point of inflection is $(1, \pi/2)$

3. B $y = \sqrt{x}$, $y' = \frac{1}{2\sqrt{x}}$. By the Mean Value Theorem we have $\frac{1}{2\sqrt{c}} = \frac{2}{4} \Rightarrow c = 1$.

The point is (1,1).

4. D $\int_0^8 \frac{dx}{\sqrt{1+x}} = 2\sqrt{1+x} \Big|_0^8 = 2(3-1) = 4$

5. E Using implicit differentiation, $6x + 2xy' + 2y + 2y \cdot y' = 0$. Therefore $y' = \frac{-2y-6x}{2x+2y}$.

When $x = 1$, $3 + 2y + y^2 = 2 \Rightarrow 0 = y^2 + 2y + 1 = (y+1)^2 \Rightarrow y = -1$

Therefore $2x + 2y = 0$ and so $\frac{dy}{dx}$ is not defined at $x = 1$.

6. B This is the derivative of $f(x) = 8x^8$ at $x = \frac{1}{2}$.

$$f'\left(\frac{1}{2}\right) = 64\left(\frac{1}{2}\right)^7 = \frac{1}{2}$$

7. D With $f(x) = x + \frac{k}{x}$, we need $0 = f'(-2) = 1 - \frac{k}{4}$ and so $k = 4$. Since $f''(-2) < 0$ for $k = 4$, f does have a relative maximum at $x = -2$.

8. C $h'(x) = 2f(x) \cdot f'(x) - 2g(x) \cdot g'(x) = 2f(x) \cdot (-g(x)) - 2g(x) \cdot f(x) = -4f(x) \cdot g(x)$

9. D $A = \frac{1}{2} \int_0^{2\pi} (\sqrt{3 + \cos \theta})^2 d\theta = 2 \cdot \frac{1}{2} \int_0^\pi (\sqrt{3 + \cos \theta})^2 d\theta = \int_0^\pi (3 + \cos \theta) d\theta$

10. A $\int_0^1 \frac{x^2}{x^2 + 1} dx = \int_0^1 \frac{x^2 + 1 - 1}{x^2 + 1} dx = \int_0^1 \left(\frac{x^2 + 1}{x^2 + 1} - \frac{1}{x^2 + 1} \right) dx = \left(x - \tan^{-1} x \right) \Big|_0^1 = 1 - \frac{\pi}{4} = \frac{4 - \pi}{4}$

11. B Let L be the distance from $\left(x, -\frac{x^2}{2} \right)$ and $\left(0, -\frac{1}{2} \right)$.

$$L^2 = (x - 0)^2 + \left(\frac{x^2}{2} - \frac{1}{2} \right)^2$$

$$2L \cdot \frac{dL}{dx} = 2x + 2 \left(\frac{x^2}{2} - \frac{1}{2} \right) (x)$$

$$\frac{dL}{dx} = \frac{2x + 2 \left(\frac{x^2}{2} - \frac{1}{2} \right) (x)}{2L} = \frac{2x + x^3 - x}{2L} = \frac{x^3 + x}{2L} = \frac{x(x^2 + 1)}{2L}$$

$$\frac{dL}{dx} < 0 \text{ for all } x < 0 \text{ and } \frac{dL}{dx} > 0 \text{ for all } x > 0, \text{ so the minimum distance occurs at } x = 0.$$

The nearest point is the origin.

12. E By the Fundamental Theorem of Calculus, if $F(x) = \int_0^x e^{-t^2} dt$ then $F'(x) = e^{-x^2}$.

13. C $\int_{-\pi/2}^k \cos x dx = 3 \int_k^{\pi/2} \cos x dx; \sin k - \sin \left(-\frac{\pi}{2} \right) = 3 \left(\sin \frac{\pi}{2} - \sin k \right)$

$$\sin k + 1 = 3 - 3 \sin k; 4 \sin k = 2 \Rightarrow k = \frac{\pi}{6}$$

14. D $y = x^2 + 2$ and $u = 2x - 1, \frac{dy}{du} = \frac{dy}{dx} \cdot \frac{dx}{du} = (2x) \left(\frac{1}{2} \right) = x$

15. B The graphs do not need to intersect (eg. $f(x) = -e^{-x}$ and $g(x) = e^{-x}$). The graphs could intersect (e.g. $f(x) = 2x$ and $g(x) = x$). However, if they do intersect, they will intersect no more than once because $f(x)$ grows faster than $g(x)$.

16. B $y' > 0 \Rightarrow y$ is increasing; $y'' < 0 \Rightarrow$ the graph is concave down. Only B meets these conditions.
17. B $y' = 20x^3 - 5x^4$, $y'' = 60x^2 - 20x^3 = 20x^2(3 - x)$. The only sign change in y'' is at $x = 3$. The only point of inflection is $(3, 162)$.
18. E There is no derivative at the vertex which is located at $x = 3$.
19. C $\frac{dv}{dt} = \frac{1 - \ln t}{t^2} > 0$ for $0 < t < e$ and $\frac{dv}{dt} < 0$ for $t > e$, thus v has its maximum at $t = e$.
20. A $y(0) = 0$ and $y'(0) = \left. \frac{\frac{1}{2}}{\sqrt{1 - \frac{x^2}{4}}} \right|_{x=0} = \left. \frac{1}{\sqrt{4 - x^2}} \right|_{x=0} = \frac{1}{2}$. The tangent line is $y = \frac{1}{2}x \Rightarrow x - 2y = 0$.
21. B $f'(x) = 2x - 2e^{-2x}$, $f'(0) = -2$, so f is decreasing
22. E $f(x) = \int_0^x \frac{1}{\sqrt{t^3 + 2}} dt$, $f(-1) = \int_0^{-1} \frac{1}{\sqrt{t^3 + 2}} dt = -\int_{-1}^0 \frac{1}{\sqrt{t^3 + 2}} dt < 0$
 $f(-1) < 0$ so E is false.
23. D $\frac{dy}{dx} = \frac{-xe^{-x^2}}{y} \Rightarrow 2y dy = -2xe^{-x^2} dx \Rightarrow y^2 = e^{-x^2} + C$
 $4 = 1 + C \Rightarrow C = 3$; $y^2 = e^{-x^2} + 3 \Rightarrow y = \sqrt{e^{-x^2} + 3}$
24. C $y = \ln \sin x$, $y' = \frac{\cos x}{\sin x} = \cot x$
25. A $\int_m^{2m} \frac{1}{x} dx = \ln x \Big|_m^{2m} = \ln(2m) - \ln(m) = \ln 2$ so the area is independent of m .

$$26. \quad C \quad \int_0^1 \sqrt{x^2 - 2x + 1} \, dx = \int_0^1 |x - 1| \, dx = \int_0^1 -(x - 1) \, dx = -\frac{1}{2}(x - 1)^2 \Big|_0^1 = \frac{1}{2}$$

Alternatively, the graph of the region is a right triangle with vertices at (0,0), (0,1), and (1,0).

The area is $\frac{1}{2}$.

$$27. \quad C \quad \int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = -\ln |\cos x| + C = \ln |\sec x| + C$$

$$28. \quad D \quad \text{Use L'Hôpital's Rule: } \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\tan x} = \lim_{x \rightarrow 0} \frac{2e^{2x}}{\sec^2 x} = 2$$

29. C Make the substitution $x = 2 \sin \theta \Rightarrow dx = 2 \cos \theta \, d\theta$.

$$\int_0^1 (4 - x^2)^{-3/2} \, dx = \int_0^{\pi/6} \frac{2 \cos \theta}{8 \cos^3 \theta} \, d\theta = \frac{1}{4} \int_0^{\pi/6} \sec^2 \theta \, d\theta = \frac{1}{4} \tan \theta \Big|_0^{\pi/6} = \frac{1}{4} \cdot \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{12}$$

$$30. \quad D \quad \text{Substitute } -x \text{ for } x \text{ in } \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x \text{ to get } \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} = e^{-x}$$

$$31. \quad C \quad \frac{dy}{dx} = -y \Rightarrow y = ce^{-x} \text{ and } 1 = ce^{-1} \Rightarrow c = e; \quad y = e \cdot e^{-x} = e^{1-x}$$

32. B $1 + 2^x + 3^x + 4^x + \cdots + n^x + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^p}$ where $p = -x$. This is a p -series and is convergent if $p > 1 \Rightarrow -x > 1 \Rightarrow x < -1$.

$$33. \quad A \quad \frac{1}{3} \int_{-1}^2 3t^3 - t^2 \, dt = \frac{1}{3} \left(\frac{3}{4} t^4 - \frac{1}{3} t^3 \right) \Big|_{-1}^2 = \frac{1}{3} \left(\left(12 - \frac{8}{3} \right) - \left(\frac{3}{4} - \frac{1}{3} \right) \right) = \frac{11}{4}$$

$$34. \quad D \quad y' = -\frac{1}{x^2}, \text{ so the desired curve satisfies } y' = x^2 \Rightarrow y = \frac{1}{3} x^3 + C$$

$$35. \quad A \quad a(t) = 24t^2, \quad v(t) = 8t^3 + C \text{ and } v(0) = 0 \Rightarrow C = 0. \text{ The particle is always moving to the right, so distance} = \int_0^2 8t^3 \, dt = 2t^4 \Big|_0^2 = 32.$$

36. B $y = \sqrt{4 + \sin x}$, $y(0) = 2$, $y'(0) = \frac{\cos 0}{2\sqrt{4 + \sin 0}} = \frac{1}{4}$. The linear approximation to y is

$$L(x) = 2 + \frac{1}{4}x. L(1.2) = 2 + \frac{1}{4}(1.2) = 2.03$$

37. D This item uses the formal definition of a limit and is no longer part of the AP Course Description. Need to have $|(1 - 3x) - (-5)| < \varepsilon$ whenever $0 < |x - 2| < \delta$.

$$|(1 - 3x) - (-5)| = |6 - 3x| = 3|x - 2| < \varepsilon \text{ if } |x - 2| < \varepsilon/3.$$

Thus we can use any $\delta < \varepsilon/3$. Of the five choices, the largest satisfying this condition is $\delta = \varepsilon/4$.

38. A Note $f(1) = \frac{1}{2}$. Take the natural logarithm of each side of the equation and then differentiate.

$$\ln f(x) = (2 - 3x) \ln(x^2 + 1); \frac{f'(x)}{f(x)} = (2 - 3x) \cdot \frac{2x}{x^2 + 1} - 3 \ln(x^2 + 1)$$

$$f'(1) = f(1) \left((-1) \cdot \frac{2}{2} - 3 \ln(2) \right) \Rightarrow f'(1) = \frac{1}{2}(-1 - 3 \ln 2) = -\frac{1}{2}(\ln e + \ln 2^3) = -\frac{1}{2} \ln 8e$$

39. D $x = e \Rightarrow v = 1$, $u = 0$, $y = 0$; $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx} = (\sec^2 u) \left(1 + \frac{1}{v^2} \right) \left(\frac{1}{x} \right) = (1)(2)(e^{-1}) = \frac{2}{e}$

40. E One solution technique is to evaluate each integral and note that the value is $\frac{1}{n+1}$ for each.

$$\text{Another technique is to use the substitution } u = 1 - x; \int_0^1 (1 - x)^n dx = \int_1^0 u^n (-du) = \int_0^1 u^n du.$$

$$\text{Integrals do not depend on the variable that is used and so } \int_0^1 u^n du \text{ is the same as } \int_0^1 x^n dx.$$

41. D $\int_{-1}^3 f(x) dx = \int_{-1}^2 (8 - x^2) dx + \int_2^3 x^2 dx = \left(8x - \frac{1}{3}x^3 \right) \Big|_{-1}^2 + \frac{1}{3}x^3 \Big|_2^3 = 27 \frac{1}{3}$

42. B Use the technique of antiderivatives by parts to evaluate $\int x^2 \cos x \, dx$

$$u = x^2 \quad dv = \cos x \, dx$$

$$du = 2x \, dx \quad v = \sin x$$

$$f(x) - \int 2x \sin x \, dx = \int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx + C$$

$$f(x) = x^2 \sin x + C$$

43. E
$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx = \int_a^b \sqrt{1 + (\sec^2 x)^2} \, dx = \int_a^b \sqrt{1 + \sec^4 x} \, dx$$

44. E $y'' - y' - 2y = 0$, $y'(0) = -2$, $y(0) = 2$; the characteristic equation is $r^2 - r - 2 = 0$.

The solutions are $r = -1$, $r = 2$ so the general solution to the differential equation is

$$y = c_1 e^{-x} + c_2 e^{2x} \text{ with } y' = -c_1 e^{-x} + 2c_2 e^{2x}. \text{ Using the initial conditions we have the system:}$$

$$2 = c_1 + c_2 \text{ and } -2 = -c_1 + 2c_2 \Rightarrow c_2 = 0, c_1 = 2. \text{ The solution is } f(x) = 2e^{-x} \Rightarrow f(1) = 2e^{-1}.$$

45. E The ratio test shows that the series is convergent for any value of x that makes $|x+1| < 1$.

The solutions to $|x+1| = 1$ are the endpoints of the interval of convergence. Test $x = -2$ and

$x = 0$ in the series. The resulting series are $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$ and $\sum_{k=1}^{\infty} \frac{1}{k^2}$ which are both convergent.

The interval is $-2 \leq x \leq 0$.